A Speculation in Theory and Practice, II

Tertium Quid

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If we take, as an indicator of the validity of a scientific/academic discipline, the ability of its exponents to disagree with one another calmly and objectively, then we must admit that the theory of music has a problem. It is a controversial field, and probably always will be; so much so that it may seem pointless to try to isolate one particular strand of controversy and trace it through a century or so of its development.

The bone of contention considered here is the question of whether the ratios 5:4 and 6:5 are acceptable as mathematical models for the major and minor third, respectively. This question is actually in some way central to most of the controversies among music theorists of all periods. The historical snapshot of the development of this question that will be examined here covers approximately 100 years, between circa 1480 and 1588. The question of interval sizes and their mathematical expression first acquires a strong relevance to the theory of tonal music during this period, and there is a fascinating reversal of positions between speculative and practical theorists some time in the late 16th century.

The background of the controversy is briefly as follows. The system of intervals called "Pythagorean", probably the most widely accepted system within the Greek and Greco-Roman traditions, recognized the octave as an equivalence relation, and the ratio 4:3 (a "perfect fourth") as the unique generating interval. Important resulting intervals were the tone 9:8 (the defect of two fourths from an octave), the ditone 81:64 (product of two tones), the limma 256:243 (defect of a ditone from a fourth), the apotome 2187:2048 (defect of a limma from a tone), and the hemiditone 32:27 (defect of a tone from a fourth). All of these ratios can be expressed using only powers of 2 and 3, the first two prime numbers. This gives the Pythagorean system an economy and consistency not shared by any of its early rivals.

A puzzling strand in Greco-Roman mathematico-musical thought is the notion of superparticular ratios (of the form N+1 : N) as a privileged class. Long before the discovery of the harmonic series, this leads to the assertion of ratios such as 5:4 for the ditone (first proposed by Archytas) and 6:5 for the hemiditone (first proposed by Eratosthenes). This, however, does not reflect an extension of the set of generators from the first two primes (2 and 3) to the first three (2, 3, and 5); at the same time, almost any superparticular ratio can be found proposed as a possible model for the limma or the apotome, including such ratios as 12:11, 15:14, 22:21, 24:23, 28:27, 36:35, and 46:45.[1]

Most of these competing tunings arose from efforts to "simplify" the division of the monochord. It is difficult to sympathize with those efforts today, when the results appear to us to be vastly more complicated than those of the Pythagorean system. Yet it is impossible to dismiss this urge to "simplify", as it is the central idea of the practical theorists of each generation, and the source of their tendency to judge the usefulness of a theoretical concept by its ease of comprehension (to some audience, or perhaps only to themselves) rather than by its internal consistency.

[1] All taken from the compendium of tetrachords in Ptolemy, Harmonics, II, Chap. 14 (Latin trans. by John Wallis, London, 1699)

One such practical theorist was Bartolomeo Ramos de Pareja (*c*. 1440-1491), whose *Musica practica* (Bologna, 1482) may be taken as the opening shot in the controversy under study here. Ramos was a thoroughgoing iconoclast, of the type (distressingly common among music theorists) with a fondness for the *argumentum ad hominem* and the beguiling assertion that all of his predecessors were wrong. He does not hesitate to take on even Boethius and Guido, two authorities who had hitherto been spoken of with unfailing respect even by those who did not agree with them in every particular. Ramos's attitude may be summed up in a single sentence: "…although [Boethius's monochord] division is useful and pleasant to theorists, to singers it is laborious and difficult to understand."[2] Ramos's own division gives the following ratios: ditone 5:4, hemiditone 6:5, tone either 9:8 or 10:9, limma 16:15, apotome 135:128. The fact that the last of these is not superparticular attracts our attention to the fact that all of these intervals can be expressed as powers of the first three prime numbers (2, 3, and 5).

Ramos states that the "consonance of the ditone" arises from its sesquiquartal (5:4) ratio. [3] This could be construed to imply that the Pythagorean ditone (81:64), not being superparticular, is not consonant. The use of the word "consonance" (*consonantia*) in the literature of this period is not sufficiently consistent to support that construction, but it is the closest that Ramos comes to saying that the two flavors of ditone might be **musically**, not only mathematically, different things.

This is the opportunity to lay out the philosophical and aesthetic implications of the music-theoretical question. Three assertions might be made:

1. 5:4 and 81:64 are two equally valid mathematical models for a single physical reality, if neither precisely accurate, both workable approximations. This would seem to obviate all mathematical discussion of music; if the only difference between 5:4 and 81:64 is the method used to find the right place for a monochord bridge, then it may as well be placed by ear.
2. 5:4 and 81:64 are distinct physical realities that nevertheless function equivalently in a musical context. This is the assumption that lies behind all temperaments: if 5:4 and 81:64 are different enough to cause trouble in closing the circle of fifths, but they are both "major thirds", then anything between them is also a "major third", and therefore a candidate for use as a compromise interval.
3. 5:4 and 81:64 are musically different, in sound and in harmonic function. If this assertion is made, then it becomes necessary to develop criteria for choosing between them within a theoretical framework that explains why one is better than the other.

Most of the charges and counter-charges that make up the controversy under study here seem to be assuming Assertion #3. The best argument in favor of the suggested construction of the Ramos quote about 5:4 being "consonant" is that if we could make Ramos say that, then he would be in agreement with Zarlino; but that is arguing from hindsight. The evidence is not there to dispute Strunk's dismissive gloss that Ramos "is himself scarcely aware of the implications of what he is advancing and claims no special virtue for his division beyond its ready intelligibility and the ease with which it can be carried out."[4]

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[2] Excerpt from *Musica practica*, trans. Oliver Strunk, in *Source Readings in Music History, Vol. II: The Renaissance* (New York: Norton, 1965), p. 11
[3] *ibid*., p. 13
[4] *ibid*., p. 10

Ramos attacked many theorists for pointless, old-fashioned complication, including the transplanted Englishman John Hothby. It is tempting to agree with Ramos after grappling with Hothby's major work, *Calopea legale*,[5] which was probably written between 1450 and 1475. Hothby attempts in this work to account for all of the chromatic notes that had by that time come into use; his explanations are obscure and circumlocutory, essentially reducing to the principle that raised notes should be sung as *mi* and lowered notes as *fa*. In this way he accounts for raised notes as far as A♯ and lowered notes as far as G♭, in a strictly Pythagorean intonation where every pitch is gotten by stacking 3:2 perfect fifths. "According to Ramos, [this] system…led to total confusion and to failure to stay on pitch…"[6] Confusion perhaps; but not to failure to hold pitch, unless that were to result from the kind of carelessness that no system is proof against. Hothby's system assigns a single unique frequency to each pitch name, whereas, as the Benedetti letters will show (*infra*), Ramos's system creates ambiguity among two or more possible frequencies for each pitch name and thus makes it impossible for singers to stay on pitch.

Hothby was galvanized by Ramos's attacks on him, replying in three treatises written in the last five years of his life.[7] Much the longest of these is the *Excitatio quædam musicæ artis per refutationem*,[8] in which Hothby offers a detailed, but disappointing, critique of Ramos's monochord division: disappointing, because Hothby's argument is simply that Ramos's ratios do not agree with Boethius's and must therefore be wrong. Hothby seems to be assuming that Ramos is making Assertion #1, that his (debatably) simpler mathematical models are acceptable alternatives for the same physical realities. Ramos, having stumbled upon something very like just intonation, has one false fifth (40:27) in his gamut; to Hothby, this is nothing special, just one more blunder in a sea of blunders. The only ground upon which Hothby defends Pythagorean ratios is that of tradition.

Hothby died in 1487, but in that same year, the controversy was continued on his behalf by his student Nicolo Burzio, in a work entitled *Musices opusculum*.[9] This may well represent the absolute nadir of rhetoric in the entire dispute, as Burzio applies a thesaurus of pejoratives to Ramos: prevaricator, impudent calumniator, worthless, arrogant, crass, mad, insolent, perverse, malicious, "father of an ox" – all these in the first three pages, and so it continues.[10] Burzio offers specific rebuttals of Ramos with almost every point that he makes; but neither, annoyingly, the question of the sizes of thirds[11] nor the division of the monochord.[12] Burzio simply restates the Pythagorean ratios with no specific references to Ramos's alternatives.

One passage requires quotation: "A ditone or major third…moves to a fifth. A semiditone or minor third, since it contains a small semitone, seeks a unison…a major sixth seeks an octave.…" Here Burzio is

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[5] ed. with French trans. by E. de Coussemaker, in *Histoire de l'Harmonie au Moyen Age* (Paris, 1852; reprint, Hildesheim: Georg Olms, 1966), pp. 297-349

[6] Albert Seay, "Bartolomeo Ramos de Pareia", in Stanley Sadie, ed.: *New Grove's Dictionary of Music and Musicians* (London: Macmillan, 1980), XV, p. 576

[7] *Tres tractatuli contra Bartholomeum Ramum*, ed. Albert Seay as Vol. X of *Corpus Scriptorum de Musica* (Rome: American Institute of Musicology, 1964)

[8] *ibid*., p. 17-57

[9] ed. & trans. Clement A. Miller, as Vol. 37 of *Musicological Studies and Documents* (Rome: American Institute of Musicology, 1983)

[10] *ibid*., p. 25-27

[11] *ibid*., p. 59

[12] *ibid*., pp. 120-125

clearly stating that thirds and sixths have tendency. He describes these interval classes as "imperfect consonances" or "compatible dissonances", the latter term borrowed from another one of his teachers, Johannes Gallicus.[13] It is immensely tempting to construe this tendency as being dependent upon the Pythagorean proportions, and thus to make Burzio and Ramos agree that the 5:4 major third is more "consonant" than the 81:64 ditone, while disagreeing over which one is more musically useful; but, again, the hard evidence simply is not there.[14]

Four years passed before Burzio's crude and intemperate assault upon Ramos brought forth a published reply, by a Ramos pupil, Giovanni Spataro, whose *Bartolomei Ramis Honesta defensio*, printed at Bologna in 1491, was not available for study here. It seems to have concluded the first phase of our controversy.

It is impossible to say, without stretching the available evidence, whether the question of the sizes of thirds was regarded at this time as a musical one or as purely mathematical. What is noteworthy is that the Pythagorean ratios are being defended by the speculative theorists, and the just ratios by the practical theorists. The Pythagorean ratios are the time-honored dogma, and the just ratios are the new, "simpler" idea, trying to knock the traditional notions off their pedestal and take their place.

The controversy did not end at this point, of course; it simply became less public for a while. It seems probable that, by the time Spataro's *Honesta defensio* was published, Franchinus Gaffurius (1451-1522) had largely completed his second major treatise, *Practica musicæ*, which was first published at Milan in 1496[15], four years after the definitive version of his *Theorica musicæ*. Irwin Young correctly observes that Gaffurius "endorsed that classical proposition that the theoretical and practical aspects of music had to be sorted out and explained separately"[16], but the true importance of that fact is that Gaffurius was one of the very few who succeeded in working both sides of the fence. It is not possible to classify him as exclusively a speculative or a practical theorist, unless in the context of some specific problem such as the present one, where we shall find him speaking primarily as a speculative theorist. The *Theorica musicæ* gives a standard Pythagorean monochord division[17]; in fact, Gaffurius adheres to the Pythagorean ratios throughout all his writings, except for one confusing passage in the *Practica*.

The passage in question is translated by Young as follows: "The sixth possesses a single harmonious mean which is a third up from the lower extreme and also sounds as the root of the diatesseron above…Generally, however, such a mediated sixth – when, that is, its mean sounds the ditone third to the lower extreme – requires that the mean pitch be lowered the tiniest bit. This is something we are taught by experiment on instruments. By a certain adjacency of sound the fourth is tempered a little toward participation in the sweet fifth, while the major third is inflected a little downward in the direction of the minor, sweeter, semiditonic third."[18]

This verbal farrago displays all of the familiar, frustrating vagueness and allusiveness of even the best writing of the period. Young, nothing daunted, draws sweeping conclusions from it: "Here Gafurius [*sic*

[13] *ibid*., p. 11
[14] *ibid*., p. 79
[15] trans. Irwin Young (Madison, WI: University of Wisconsin Press, 1969)
[16] *ibid*., p. xviii
[17] Book V, Chapter IV
[18] *Practica*, pp. 127-128

et semper] acknowledges the need to temper the sizes of each of the…intervals which make up a composite major sixth (*i.e*., a first-inversion minor triad) in order to accommodate the sense of auditory pleasure, attesting his awareness of the practical world of music about him. He attained his objective by the simple expedient of lowering the mean (*i.e*., the fifth of the triad) by the distance of an 81"80 (Didymic) comma. Thus Pythagoras's 81:64 major third was diminished into the 'sweeter' 5:4 interval, while the fourth was expanded into a 27:20 interval, approaching the 'sweeter' fifth."[19] It is, of course, impossible to either prove or disprove that this is the intended sense of the passage. Young appears to be reasoning from hindsight in making Gaffurius describe the false supertonic triad of the just major scale.

Young provides additional glosses[20] that try to make Gaffurius anticipate just intonation. It would have been highly interesting and convenient if Gaffurius had actually meant what Young alleges that he meant, but, in sober fact, Gaffurius must remain solidly in the Pythagorean camp.

It was Gaffurius who, in 1520, reactivated the controversy in public, with the publication of his *Apologia adversus Johannem Spatarum*,[21] which appears to be an escalation of a private quarrel between Gaffurius and Spataro of some years' standing.[22] The *Apologia* is not couched in such brutally insulting language as was Burzio's *Musices opusculum*; it is more sardonic and dismissive. Its marginal rubrics are a roll-call of classical authority, flagging citations from Boethius, Guido, Bakcheios the Elder, Isidore of Seville, Pythagoras, Plato, Aristotle, Ptolemy – and Ramos, this last only to be patiently corrected. Gaffurius's arguments are essentially two:

- Ramos and Spataro assert that their thirds are musically equivalent to Pythagoras's and only mathematically different (*i.e*., Assertion #2 above); but in so saying, they are wrong.
- the just ratios recommended by Ramos and Spataro are not original with them, but had been proposed by Ptolemy in his Harmonics.

The second of these arguments is trivial and will not be considered further; but the first one is the most sophisticated argument that we have found any party to the controversy making thus far.

Gaffurius is careful to only use Pythagorean terminology (ditone, semiditone) to refer to Pythagorean ratios. He states that 6:5 is a "minor third" that exceeds the semiditone by 81:80. Later, discussing the just major triad 6:5:4, he states that it cannot be reconciled with the classical description of the fifth as being made up of three tones and a limma, because the sesquiquarta (5:4) falls short of a ditone, and repeats that the sesquiquinta (6:5) is greater than a semiditone.[23]

The question of semitones is even more interesting: "*In secunda tua detractoria latratione obsignata Bononiæ die .xxii.Marti.1519. afferis hoc minus semitonium .256.ad.243. illud precise non esse quod usui evenit in consonantiis novi instrumenti harmonici: sed hoc majus atque illo esse proportione*

[19] *ibid*., p. 128n
[20] *ibid*., pp. xxii-xxiii
[21] Turin; reprinted in facsimile in *Monuments of Music and Music Literature in Facsimile*, Second Series, No. XCVI (New York: Broude Bros., 1979)
[22] *ibid*., pp. (11-12), where Gaffurius refers to letters from Spataro to himself dated 28 February 1519 and 22 March 1519.
[23] *ibid*., p. (7)

sesquioctogesima."[24] Here Gaffurius quotes Spataro as saying in his letter that the semitone used on instruments is not the Pythagorean limma (256:243), but an interval that is larger by 81:80 (therefore 16:15). Gaffurius acknowledges the truth of this, although without touching upon the question of whether the two semitones are musically equivalent. He then proceeds to fog the issue by citing a flurry of Ptolemy's even stranger semitones (as mentioned above), including 15:14, 28:27, 24:23, and 46:45. He seems to be arguing against superparticularity as a criterion for semitones when he points out that the Pythagorean ration 256:243 is not superparticular but superpartient. The argument reduces to the same basic (and rather weak) justification that Gaffurius offers in the case of the thirds: Pythagorean ratios are better because they are traditional.

There is no attempt to characterize the sound of any of these things. Gaffurius does speak at one point of a *suavissimum medietatem*, but this is the proportion 6:4:3, being contrasted with 6:5:3. We may wish that he had elaborated on his point about the musical distinction between 5:4 and 81:64.

Spataro's reply took the form of a brief treatise entitled "*Dilucide et probatissime demonstratione…*" or (in full) "Clear and Fully Verified Proofs, by Master Johannes Spataro, Musician of Bologna, in Response to Certain Vain and Frivolous Apologies Brought Forth by Franchino Gafurio, Master of Mistakes".[25]

Amid a good deal of tedious wrangling, there are occasional flashes of insight, as when Spataro says that the ratio 5:3 "represents the major sixth as it is used in real music",[26] the earliest instance found here of a justification of just intervals in musical rather than purely mathematical terms. Yet later Spataro displays a remarkable confusion when he faults Gaffurius for deriving intervals from the proportion 6:4:3, omitting 5 instead of using the complete arithmetical sequence 6:5:4:3:2[27] (here is the *senario*, 37 years before the publication of Zarlino's *Istitutioni*). After a long previous discussion of primality and relative primality, Spataro ought to be able to realize that Gaffurius is excluding 5, not as a member of an arithmetical series, but as a prime factor.

The second phase of the controversy closes here, in 1521, with the positions essentially unchanged from 1491 but more clearly stated. The Pythagorean ratios still have the entrenched position of authority, owing to their long history, but they are coming under increasing attack from the just ratios, which are beginning to be recognized as representing different sounds and are being claimed as better on that basis.

It remained to devise a mathematical, symbolic, aesthetic, and philosophical framework for the just intervals that could compete in credibility with that of the Pythagorean intervals. This was the goal of Gioseffo Zarlino (1517-1590), whose *Istitutioni harmoniche*[28], though probably not primarily intended as a polemic, opens the third phase of the controversy.

The Pythagorean symbolism made much use of the first four natural numbers (the *tetractys*) and their sum, 10. Only those intervals were defined as consonances whose ratios could be expressed in the superparticular genus within the *tetractys*: the octave 2:1, the fifth 3:2, and the fourth 4:3. The latter

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[24] *ibid*., p. (12)
[25] Bologna, 1521; facsimile, with German trans. and commentary by Johannes Wolf, as No. 7 of *Veröffentlichungen der Musik-Bibliothek Paul Hirsch* (Berlin: Martin Breslauer, 1925)
[26] *ibid*., p. (5)
[27] *ibid*., p. (8)
[28] First ed., Venice, 1558; second ed., revised and enlarged, 1573

two, along with their difference the tone (9:8), were added and subtracted to generate all other intervals. The result was a system in which any interval could be expressed as $3^x:2^y$, where the exponents x and y could be any integer (positive, negative, or zero). Another way to express this is to say that the system is defined by an **equivalence** relation, 2:1, and a **generator**, 3:2. The important point is that there is only one generator, and it is the most economical possible, involving only the first two prime numbers, 2 and 3.

Zarlino (and the theorists of the next four centuries, who adopted his mathematics almost unanimously) expanded the privileged set of natural numbers from four to six: the *tetractys* was supplanted by the *senario*. Again, only those intervals that are superparticular within the *senario* are deemed fully consonant: the three Pythagorean consonances plus the major third 5:4 and the minor third 6:5. This results in a system where any interval can be expressed as $(3^x+5^z):2^y$, where again, any of the exponents can be any integer. The important point here is that there are now two generators, 3:2 and 5:2. The result is that there is no longer a one-to-one correspondence between musical symbol (pitch-class name) and physical model. For any interval, we must decide whether to find its ratio by stacking 3:2 fifths, or 5:4 thirds, or which combination thereof. Since the powers of 3 and the powers of 5 can never coincide, the resulting ratios will be different in each case.

Zarlino found some "traditional" authority for these ideas in one of Ptolemy's tetrachords, the *diatonon syntonon*. This tetrachord is divided into the superparticular ratios 9:8, 10:9, and 16:15. Unlike most of Ptolemy's tetrachords, all of these ratios can be formed from the first three prime factors, 2, 3, and 5; therefore they generate sum-products that fall within the *senario* (10:9 x 9:8 = 5:4; 9:8 x 16:15 = 6:5).

It is interesting to see to what lengths Zarlino goes to erect this mathematical framework for the just thirds. Evidently the argument from euphony is not sufficient; it is not good enough to say that a thing sounds good, reasons must also be given why it ought to sound good. It is also notable that Zarlino is propounding Assertion #2, *viz*. that intervals that are physically and mathematically different may yet be musically equivalent, as for example a major second may be either 9:8 or 10:9 and an augmented unison (chromatic semitone) may be either 25:24 or 135:128.

This mutability of intervals is a double-edged sword. On one hand, it fits nicely with Zarlino's philosophical distinction between vocal music (*musica naturale*) and instrumental music (*musica artificiata*), according to which the latter can only approximate the perfection of the former. Chapters 41-45 of Part II of the *Istitutioni* set forth Zarlino's belief that instruments, made by fallible human hands, cannot possibly produce the pure and accurate consonances of voices singing together, but they may come tolerably close, especially with the aid of temperament ("participation", "distribution"), of which he describes a few varieties in detail. But this problem is crucially exacerbated by the other consequence of mutable intervals, which is that it is no longer possible to model intervals uniquely and therefore to identify pitches unambiguously. This leads to the phenomenon of changing pitch in *a cappella* vocal polyphony, as described by the mathematician Giovanni Battista Benedetti (1530-1590) in his letters to Cipriano da Rore, *circa* 1563.[29]

[29] Published by Benedetti in his *Diversarum speculationum…* (Turin, 1585); discussed by Claude V. Palisca in "Scientific Empiricism in Musical Thought", in Hedley Howell Rhys, ed., *Seventeenth Century Science and the Arts* (Princeton: Princeton University Press, 1961), pp. 104*ff*

The Benedetti letters point up a great change in the terms of the controversy, a change that quietly took place some time after the publication of the first edition of Zarlino's *Istitutioni*. The apparent simplicity of the *senario* system has enabled it to oust the Pythagorean system from its position of authority, almost without a struggle. Those theorists who are arguing against just intonation are now the carping insurgents outside the gates, and most of them are no longer even arguing for the Pythagorean system itself, but for some form of temperament that might ideally combine some of the best features of each.

The simplicity of the *senario* system is qualified above as "apparent" because, although (say) 5:4 certainly looks like a "simpler" ratio than 81:64, the *senario* system, employing as it does the first three prime factors, is thereby more complex and ambiguous than the Pythagorean system, which employs only the first two prime factors. This contrast does not leap off Benedetti's pages, as his argument against just intonation bristles with ratios like 27:25, 80:81, 32:27, or 25:24. These look more complicated than Zarlino's explanations, but they also look comparable to Pythagorean ratios such as 81:64, 256:243, or 2187:2048. The real story is told by the prime factorizations. If 32:27 is expressed as (2x2x2x2x2):(3x3x3x3), and 6:5 as (2x3):5, then it becomes apparent that 6:5 inhabits a more complex system, as it involves a larger number of distinct prime factors.

The only music theorist of Zarlino's generation who continued specifically to defend the Pythagorean system was not primarily a music theorist at all. Girolamo Mei (1519-1594) was primarily a student of Greek civilization and culture; as such, he acquired a familiarity with the Greek literature on music that far surpassed that of any of his contemporaries. Mei conducted an extensive correspondence[30] with Zarlino's sometime student Vincenzo Galilei (*c*. 1520-1591) between 1572 and 1581, covering many specialized topics in Greek theory. Mei argues for Pythagorean ratios on the familiar ground of tradition[31], but also – uniquely, as far as discoverable here – on grounds of sound and current practice: "Stretch out…two…strings of equal length and width and [place] frets under them accurately according to the…two species of tuning – syntonic (*i.e.*, *diatonon syntonon*) and diatonic (*i.e.*, *diatonon ditonaion*, the Pythagorean tetrachord) – and then…observe which of the two strings gives the notes that correspond to what is sung today."[32]

If Galilei tried this experiment, it seems to have convinced him that neither the *diatonon syntonon* nor the *diatonon ditonaion* was fit to serve as sole model. Discussion of this question takes up the first major section (nearly one-third) of Galilei's *Dialogo della musica antica et della moderna*.[33]

Galilei's essential premises are:

- that the senario system is fatally flawed by the ambiguities of interval size and pitch discussed above;
- that the Pythagorean system is mathematically consistent but also cumbersome, and its thirds and sixths are dissonant;
- that therefore some kind of temperament is the only way to go.

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[30] ed. with commentary by Claude V. Palisca, as Vol.3 of *Musicological Studies and Documents* (Rome: American Institute of Musicology, 1960)
[31] *ibid*., pp. 63-67
[32] trans. Palisca, *ibid*., p. 67; original, p. 140
[33] Florence, 1581; trans. with commentary by Robert H. Herman (unpub. Ph.D. dissertation, North Texas State Univ., 1973)

He develops his argument as follows: the existing theoretical discussions are flawed and conflict with each other; the major discrepancy is in the mathematics of the diatonic octave; the *diatonon syntonon* is chief among models – a position that it has gained only recently due to the advocacy of "practical musicians" following Zarlino, and that was formerly held, since the most ancient times, by its principal rival, the *diatonon ditonaion*.

But, says Galilei, the allegation that the *diatonon syntonon* is the system universally used in contemporary music is implausible in light of its internal contradictions, arising from its two unequal tones (9:8 and 10:9) and the resulting, numerous mutable pitches.

Galilei is extremely careful with his terminology, using interval names of the modern quality-quantity type only for just intervals and Greek names for the Pythagorean intervals. This indicates that he regards them as musically different, a supposition that is confirmed by his repeated statements that the Pythagorean "thirds" (the ditone and semiditone) are dissonant.

Galilei takes every opportunity to insult and chastise the "practical musician", whose incurable superficiality leads him to trade the apparent complexity of the Pythagorean system for the apparent simplicity of the *senario*.

Zarlino responded to Galilei's criticisms with his *Sopplementi musicale* (Venice, 1588), in which he completely evades the question of mutable intervals and pitches. This, in fact, marks the end of the controversy, if not of the polemics; charges and counter-charges continue, with such works as Galilei's *Discorso intorno alle opera di G. Zarlino* (Florence, 1589), but nothing further of substance is said on either side.

In conclusion, an interesting afterecho of this century of verbal strife deals with some of the purely musical implications of its progress and outcome. D. P. Walker, discussing Johannes Kepler, writes: "Kepler believed, with the majority of competent scholars, that ancient music…was not polyphonic in any way resembling modern music, and that this difference was reflected in the prevailing system of intonation: Pythagorean (in which the thirds and sixths are dissonant) for the ancients, and just (in which they are consonant) for the moderns. …[A]ll systems of intonation…are mathematical ideals…even an approximation is much more difficult to achieve in just intonation than in Pythagorean…because…just intonation is hopelessly unstable even in the simplest diatonic music… For music which is monodic, or in which the interest is concentrated on melody, Pythagorean intonation is more suitable than just, since…the very narrow semitones give greater sharpness to the shape of the melody. For polyphonic music such as that of the sixteenth to the nineteenth centuries, in which the major triad occupies a dominating and central position, just intonation has the advantage of making this chord as sweet as possible…though [just intonation] has the disadvantage of…instability of pitch, of unequal tones, and of much wider semitones. These remarks are borne out by the history of Western music. …[W]ith the full development of polyphony in the later Middle Ages theorists began to accept thirds and sixths as 'imperfect consonances', though still giving the Pythagorean ratios. …[T]heir major triads would be no more consonant than their minor triads; and in fact it is not until the later sixteenth century that harmony begins to be dominated by the major triad, as opposed to the minor…"[34]

[34] D. P. Walker, "Kepler's Celestial Music", in the *Journal of the Warburg and Courtauld Institute*, Vol. XXX (1967), pp. 229-231

Assertions of causality in so fluid a thing as the development of an art form are inherently risky. Walker articulates widely shared beliefs, based on the assumption that theory follows practice. The converse is also possible: practice may follow theory. It is entirely plausible that the "dominating and central position" of the major triad in music of the common-practice period may have resulted from the adoption of just intonation, and that that adoption may have been originally impelled by the deceptive elegance of its mathematics – subsequently, from the 18th century, reinforced by the newly-discovered harmonic series. It is harder to follow Walker's implication that the interest of "the polyphonic music…of the sixteenth to the nineteenth centuries" is not concentrated on melody; indeed this is very nearly a *contradictio in adjecto*. The commonplace assertion that Pythagorean thirds, sixths, and triads are unsuitable for polyphonic music, as well as for most styles of homophonic music, is actually a *non sequitur*. From *c*. 1675 at any rate, the statistical dominance of major-triad sonorities is greatly due to secondary dominant chords, which are chromatically altered from minor quality. These triads, by any common-practice criteria, are dissonant simply because they are altered.

In any case, the victory of just intonation as a theoretical and mathematical model was complete by the end of the 16th century, even though it proved impracticable as a performance model in its pure state. Pythagorean intonation was cast into a disrepute from which it may only now be beginning to recover. And music theorists remain, on the whole, unable to disagree with one another calmly or objectively. In the face of these damaging facts, and of the doubt that they cast upon the usefulness of studying the controversies of the past, it is necessary to remember that, as music is an art (and therefore includes a science), there is no such thing as final proof: no question can ever be settled so definitively as to preclude its being reopened at any future time.

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